Influence Dynamics on Social Networks

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August 28, 2014

Outline

1. Introduction
2. Influence Maximization: Linear Threshold Model
3. Fluid Limit Characterization: Linear Threshold Model
4. Coevolution of Influence and Content in Mobile P2P Networks
5. Competition over Timelines

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Social Networks

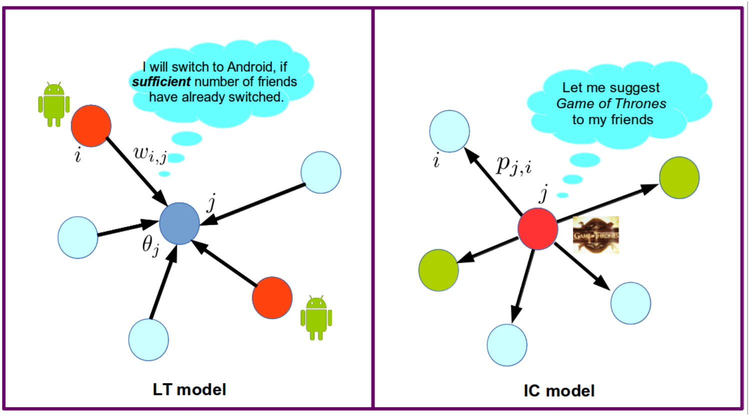
- Networks are all-pervasive
  - **Natural:** protein interaction networks, food webs, neuronal networks
  - **Man-made:** transportation networks, power grids, Internet
- Social networks: individuals as nodes, and relationships as edges
- Edges can be directed, weighted
- Some examples: email-contact networks, co-authorship networks
- Online social networks: Massive active user base; Facebook (>1billion), Twitter (>250million)

Social Network Analysis

- Several structural properties have been empirically observed
  - **small-world:** small average path length, *six degrees of separation*
  - **high clustering coefficient:** large number of triangles
  - **scale-free:** power law degree distribution
- Generative models: Preferential attachment, Watts-Strogatz model
- Network dynamics: evolution of edge structure, communities
- Focus of this thesis: **Analytical approach to dynamics on networks**
  - Characterizing cascades on networks
  - Spread of influence, information, epidemics, node-failures

Influence spread models

- Threshold model for collective behavior [Granovetter 1978]
- Maximizing influence spread (viral marketing) [Kempe et al. 2003]
- Two widely used models:
  - Linear Threshold (LT) model, Independent Cascade (IC) model



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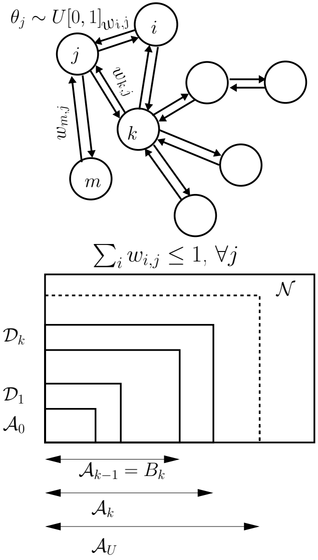
Linear Threshold Model

Introduced by Kempe et al. [2003]

- The social network is modeled by a directed weighted graph $\mathcal{N} = (V, E)$, with edge weights $w_{i,j}$, $\sum_i w_{i,j} \leq 1$
- At the beginning, each node j independently chooses a random threshold $\theta_j \sim U[0,1]$
- Begin with initial set of *seed* nodes $\mathcal{A}_0$
- At step k, a node j gets activated if, it had been inactive until step $k-1$ and

$$\sum_{i \in \mathcal{A}_{k-1}} w_{i,j} \geq \theta_j$$

- The process stops when a *terminal set* $\mathcal{A}_U$ is reached
- **Expected Influence:** $\sigma^{(\mathcal{N},\mathcal{A}_0)} = \mathbb{E}^{(\mathcal{N},\mathcal{A}_0)}[|\mathcal{A}_U|]$



Influence Maximization: Greedy Algorithm

Picking the most influential set of K nodes

$$\max \sigma^{(\mathcal{N},\mathcal{A}_0)}$$

$$\text{s.t. } \mathcal{A}_0 \subset \mathcal{N}$$

$$|\mathcal{A}_0| = K$$

- The problem is NP-hard: equivalent to *Vertex cover*
- $\sigma^{(\mathcal{N},\mathcal{A}_0)}$ is monotone and submodular (i.e., diminishing returns) in $\mathcal{A}_0$
- Network heuristics (degree centrality, distance centrality, etc.) perform poorly
- Greedy hill-climbing algorithm: $(1 - \frac{1}{e})$ approximation guarantee, computationally expensive
- Other efficient algorithms/heuristics: CELF [Leskovec et al. 2007], LDAG [Chen et al. 2010]
- Our approach: **Leverage analytical insights from the LT model**

Activation probabilities

$g_j^{(\mathcal{N},\mathcal{A}_0)}(k)$: probability that node j
gets activated in step k,
starting with seed set $\mathcal{A}_0$ in network $\mathcal{N}$

$$\sigma^{(\mathcal{N},\mathcal{A}_0)} = \sum_{j \in N} g_j^{(\mathcal{N},\mathcal{A}_0)}$$

Lemma

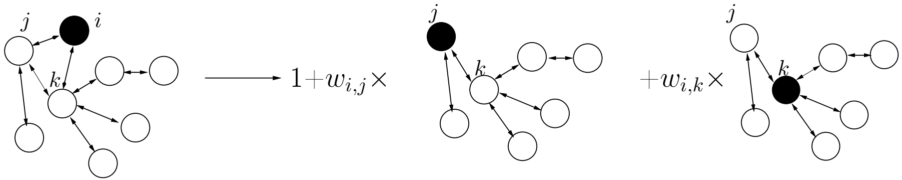
1. $j \in \mathcal{A}_0$,
   1. $g_j^{(\mathcal{N},\mathcal{A}_0)}(0) = 1$
   2. $g_j^{(\mathcal{N},\mathcal{A}_0)}(k) = 0$, *for all* $k > 0$
2. $j \notin \mathcal{A}_0$,
   1. $g_j^{(\mathcal{N},\mathcal{A}_0)}(0) = 0$
   2. $g_j^{(\mathcal{N},\mathcal{A}_0)}(k) = \sum_{l \in \mathcal{N} \setminus \{j\}} g_l^{(\mathcal{N} \setminus \{j\},\mathcal{A}_0)}(k-1)\, w_{l,j}$, *for all* $k > 0$

Total Expected Activation due to $\{i\}$: $\sigma^{(\mathcal{N},i)}$

Theorem

Given a social network $\mathcal{N}$, with influence matrix $\mathbf{W}$, the total influence of any node i in the network under the LT model is given by

$$\sigma^{(\mathcal{N},i)} = 1 + \sum_{j \in \mathcal{N} \setminus \{i\}} w_{i,j} \sigma^{(\mathcal{N} \setminus \{i\}, j)} \tag{1}$$



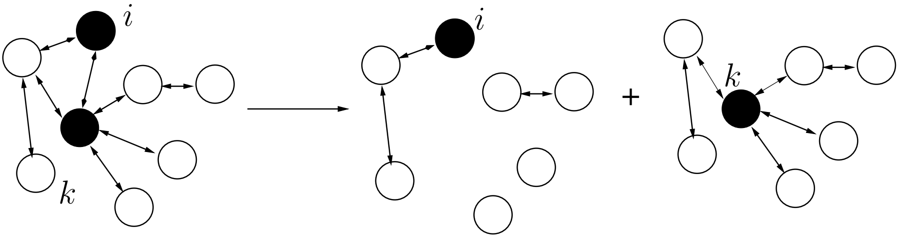
Singleton Initial Set $\{i\}$

Total Expected Activation due to Set $\mathcal{A}_0$: $\sigma^{(\mathcal{N},\mathcal{A}_0)}$

Theorem

Given a network $\mathcal{N}$ with influence matrix $\mathbf{W}$ and an initial set $\mathcal{A}_0$, for all $i \in \mathcal{A}_0$, define sub-networks $\mathcal{N}_i^{\mathcal{A}_0} = \{\mathcal{N} \backslash \mathcal{A}_0\} \cup \{i\}$.
Then the expected influence of the initial set $\mathcal{A}_0$ is given by,

$$\sigma^{(\mathcal{N},\mathcal{A}_0)} = \sum_{i \in \mathcal{A}_0} \sigma^{(\mathcal{N}_i^{\mathcal{A}_0}, i)} \tag{2}$$

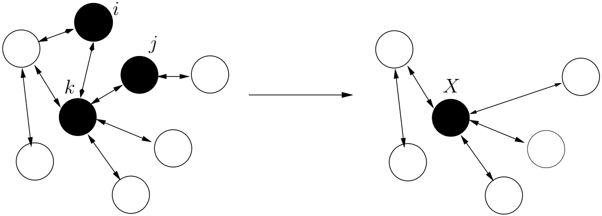


Initial Set $\mathcal{A}_0 = \{i, k\}$

Replacing an Initial Set with a "Supernode"

Theorem

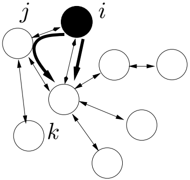
Given a network $\mathcal{N}$, with influence matrix $\mathbf{W}$, an initial set $\mathcal{A}_0$ can be replaced by a supernode X as shown below. Then, $\sigma^{(\mathcal{N},\mathcal{A}_0)}$ is equal to the influence of X in the modified network, with X being counted as $|X|$ nodes instead of 1.



Replacing the initial set with a supernode

Equivalence to Markov chains: Insights into Pagerank

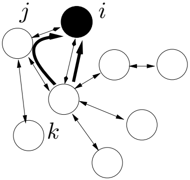
- Reverse the edges in the social network under LT model to yield a Markov chain
- Influence spread $\sim$ acyclic path probabilities (APP) to the initial set
- Provides insights into performance of Pagerank
    - Pagerank performs better in graphs with lesser cycles



$g_k^{(\mathcal{N},i)} = w_{i,k} + w_{i,j} \times w_{j,k}$

Social Network

$$\xrightarrow{p_{i,j} = w_{j,i}}$$



$c(k \to i) = p_{k,i} + p_{k,j} \times p_{j,i}$

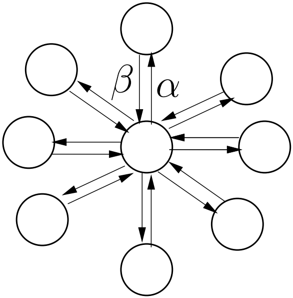
Equivalent Markov Chain

Toy Networks

Star topology

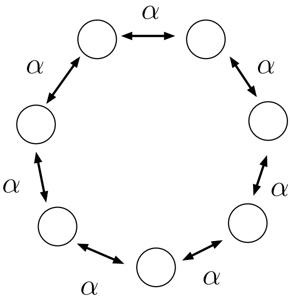
- Edge weights: $\alpha(\leq 1)$ and $\beta(\leq \frac{1}{N-1})$
- $\exists\ \alpha^\star$ such that for $\alpha < \alpha^\star$, hub does not feature in the optimal initial set

$$\alpha^\star = \frac{K}{(N-K)\frac{1}{\beta} - K(N-K-1)}$$



Ring topology

- $\alpha \leq 0.5$: influence of any node on each of its neighbours
  $\sigma^{(\mathcal{N},A(K))} = K + 2\frac{\alpha}{1-\alpha}\left(K - \sum_{i=1}^{K} \alpha^{l_i}\right)$
- Optimal seed: Uniformly spaced K nodes
- With $\alpha = 0.5$, for large N and small K, the influence of $A(K)$ grows as $3K$

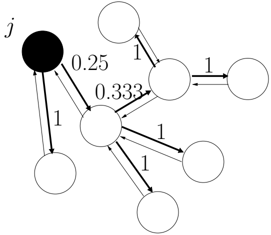


Degree Based Model: General Result on Influence

Theorem

Consider an acyclic undirected network $\mathcal{N}$. For $(i,j) \in E$, define $w_{i,j} = \frac{1}{d_j}$, where d_j is the degree of node j. Then, for any node $i \in \mathcal{N}$,

$$\sigma^{(\mathcal{N},i)} = d_i + 1$$

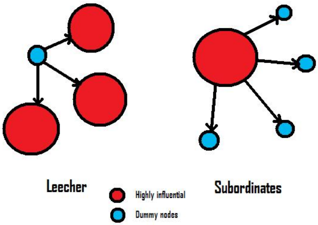


$$\sigma^{(\mathcal{N},j)} = 1 + 1 + 0.25(1 + 1 + 1 + 0.333(1 + 1 + 1)) = 3 = d_j + 1$$

- A node's local property (degree) governs its global property (influence on the entire network)

Influence Maximization: Improving the Greedy Algorithm

- Greedy algorithm: kth stage involves influence computation for $|\mathcal{N}| - k + 1$ sets of size k and this is repeated for K stages
- **G1**: Nodes listed in decreasing order of individual influences
- Top k nodes in **G1** not effective because it may contain *dummy* nodes
- Let $\mathcal{X}$ denote the set of nodes in **G1** which are above i and have been picked to be part of the initial set
- A node i is a ***leecher*** to set $\mathcal{X}$, if $\sigma^{(\mathcal{N},i)} \approx \sum_{j \in \mathcal{X}} w_{i,j} \sigma^{(\mathcal{N} \setminus i,j)}$
- A node i to be an α-***subordinate*** of set $\mathcal{X}$ if $g_i^{(\mathcal{N},\mathcal{X})} > \alpha$



G1-Sieving Algorithm

Evaluate $\sigma^{(\mathcal{N},i)}$ for all $i \in \mathcal{N}$

Sort nodes in decreasing order of $\sigma^{(\mathcal{N},i)}$ to get G1

$\mathcal{X} = G1(1)$, $G1 = G1 \backslash \mathcal{X}$

for $c = 2$ *to* K **do**

  if $G1 = \emptyset$ **then**

    break

  end

  for $i \in G1$ **do**

    if $g_i^{(\mathcal{N},\mathcal{X})} > \alpha \parallel \sigma^{(\mathcal{N}\backslash\mathcal{X},i)} < \epsilon$ **then**

      $G1 = G1 \backslash \{i\}$

      continue

    end

    Evaluate $\sigma^{(\mathcal{N}\backslash\mathcal{X},i)}$

  end

  $v = \arg\max_{i \in G1} \sigma^{(\mathcal{N}\backslash\mathcal{X},i)}$

  $\mathcal{X} = \mathcal{X} \cup v$, $G1 = G1 \backslash \{v\}$

end

- $\mathcal{R}$: set of all papers (excluding single-authored)
- $\mathcal{N}$: union of all authors of the papers in $\mathcal{R}$
- $\forall\ r \in \mathcal{R}$, n_r: number of coauthors for paper r
- $\delta(i, r)$: Indicator for author i being a co-author of paper r
- Strength of collaboration between authors i and j

$$\tilde{w}_{i,j} = \sum_{r \in \mathcal{R}} \frac{\delta(i, r)\delta(j, r)}{n_r - 1}$$

- The influence matrix W is then obtained by normalizing $\tilde{W}$.
- The simulations in the following sections are carried out on a coauthorship network of NetScience community containing 1589 nodes.

Performance of G1-sieving

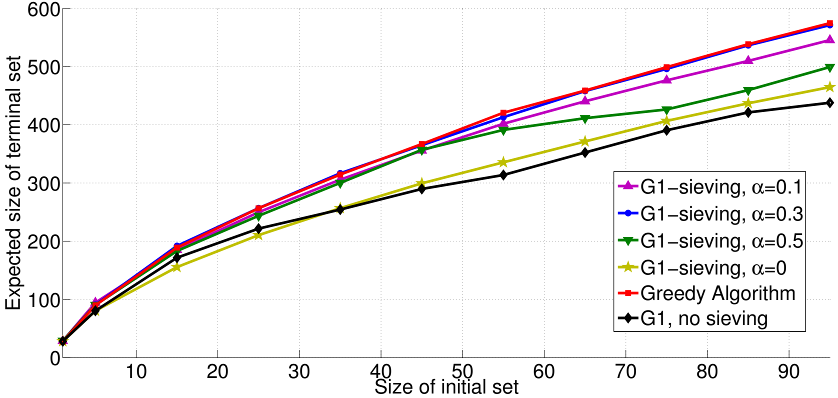


Figure: Comparison of various sieving thresholds with Greedy algorithm

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Technology Adoption

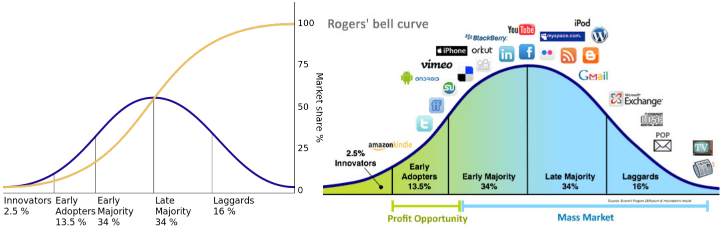


Figure: Everett Roger's bell curve shows adoption of an innovation over time

- bell-shaped curve: incremental change in adoption
- S-shaped curve: cumulative adoption
- Early adopters (innovators) $\sim$ least threshold for adoption
- "bell curve" does not imply normally distributed threshold

General threshold distribution [Granovetter 1978]

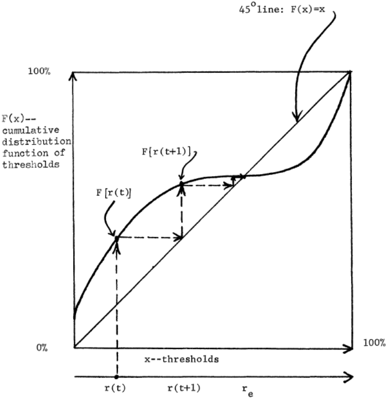
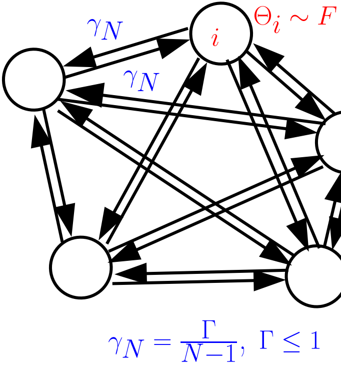


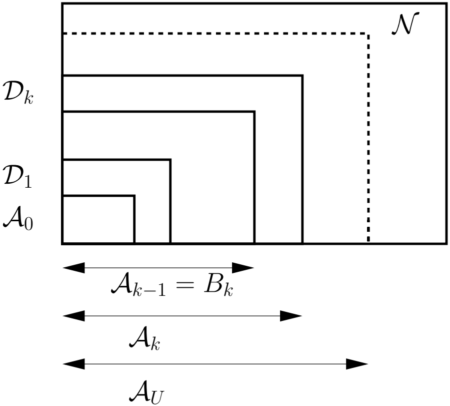
Figure: Granovetter's Fixed point dynamics $r(t+1) = F(r(t))$. Intersections indicate equilibrium points.

Issues with Granovetter's dynamics

- Thresholds are resampled: influence spread is not *progressive*
- Predicts no spread for uniform distribution

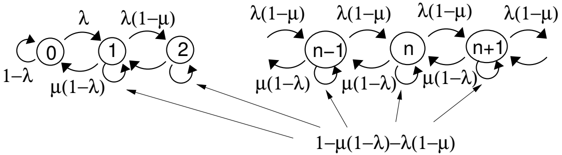
HILT model



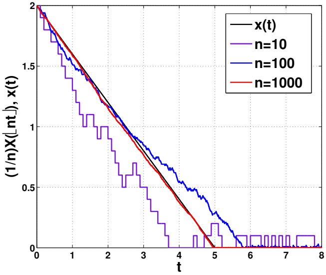


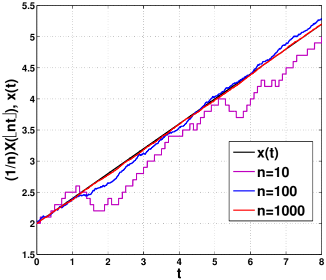
- Complete network on $\mathcal{N}$, edge weights $\gamma_N = \frac{\Gamma}{N-1}$
- Random threshold $\Theta_i \sim F$, $i \in \mathcal{N}$
- $A(k) = |\mathcal{A}(k)|$, $D(k) = |\mathcal{D}(k)|$ and $B(k) = |\mathcal{B}(k)|$
- $(B(k), D(k))$ is a Markov chain
- Kurtz theorem [1970]: Convergence of Markov chains to o.d.e.s

Fluid Limits - Kurtz Theorem



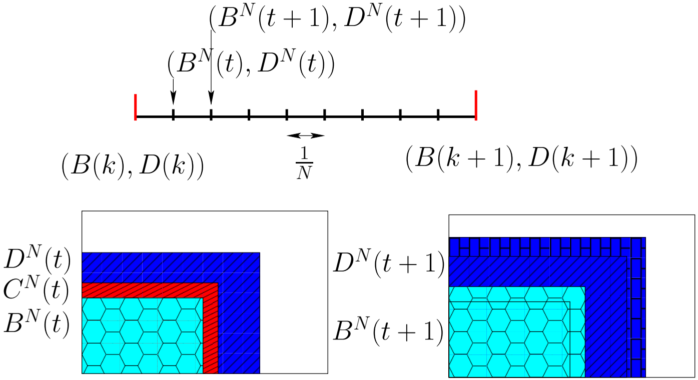
- Queue length process, $X(k), k \geq 0$, $\bar{X}^{(N)}(t) := \frac{1}{N}X(\lfloor Nt \rfloor)$
- ODE: $\dot{x}(t) = (\lambda - \mu)I_{\{x(t)>0\}}$ with $x(0) = \bar{X}^{(N)}(0)$, for each N





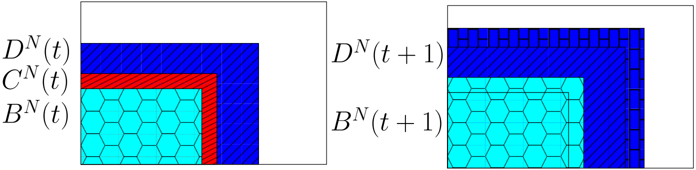
Scaling and Drift Equations

Scaled Markov process $(B^N(t), D^N(t))$ evolving over mini-slots



- Each infectious destination in $D^N(t)$ attempts to influence the relays with probability $\frac{1}{N}$
  - Success: Contributes its influence of $\frac{\Gamma}{N-1}$ and moves to $B^N(t+1)$
  - Failure: Stays in the $D^N(t+1)$

Scaling and Drift Equations



$$C^N(t) = \frac{D^N(t)}{N} + Z_b^N(t+1)$$

$$B^N(t+1) = B^N(t) + C^N(t)$$

$$\begin{aligned} D^N(t+1) &= D^N(t) + \mathbb{E}\left[\frac{F(\gamma_N(B^N(t)+C^N(t))) - F(\gamma_N B^N(t))}{1 - F(\gamma_N B^N(t))}\right] \times \\ & \left(N - B^N(t) - D^N(t)\right) - C^N(t) + Z_d^N(t+1) \end{aligned}$$

Fluid Limit for Interest Evolution - HILT Model

- Defining $\tilde{B}^N(t)$, $\tilde{C}^N(t)$ and $\tilde{D}^N(t)$ as the fractional processes, we can then state the following theorem

Theorem

Given the interest evolution Markov process $(\tilde{B}^N(t), \tilde{D}^N(t))$*, for the threshold distribution with density* $f(\cdot)$*, with bounded* $f'(\cdot)$ *and hazard function* $h_F(x) = \frac{f(x)}{1-F(x)}$*, we have for each* $T > 0$ *and each* $\epsilon > 0$*,*

$$\mathbb{P}\left(\sup_{0 \le u \le T} \left|\left|(\tilde{B}^N(\lfloor Nu \rfloor), \tilde{D}^N(\lfloor Nu \rfloor)) - (b(u), d(u))\right|\right| > \epsilon \right) \overset{N \to \infty}{\to} 0$$

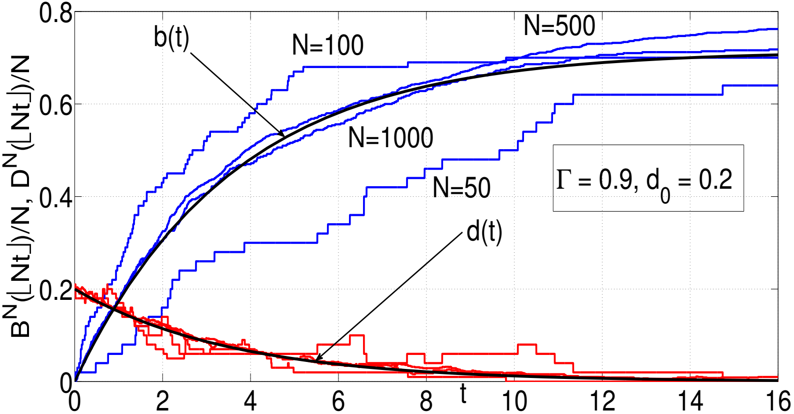
where $(b(u), d(u))$ *is the unique solution to the o.d.e.,*

$$\dot{b} = d \tag{3}$$

$$\dot{d} = h_F(\Gamma b)\Gamma d(1 - b - d) - d \tag{4}$$

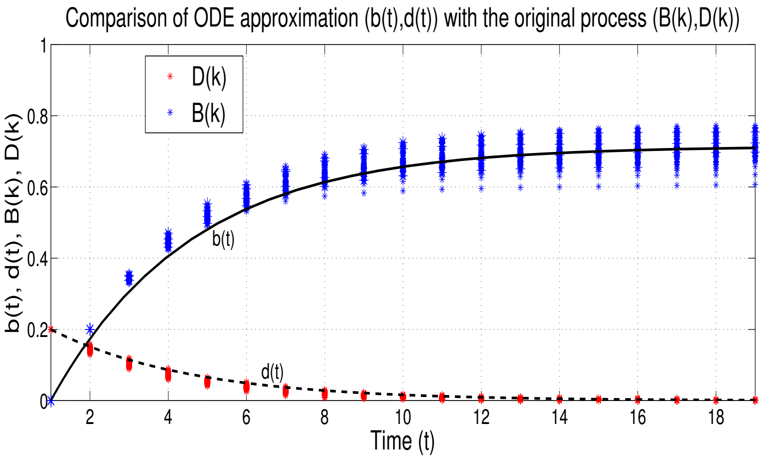
with initial conditions $(b(0) = 0, d(0) = d_0)$.

Convergence of the Scaled HILT Markov Chain to the ODE



- Influence threshold distribution is Uniform$[0, 1]$

Comparison of ODE with Unscaled HILT Markov Chain



- $N = 1000$
- We can conclude that the approximation works well for populations of a few 1000s

HILT Model: Effect of Threshold Distribution

- **Uniform threshold distribution:** $h_F(x) = \frac{1}{1-x}$. Let $r = 1 - \Gamma + \Gamma d_0$ then we have,

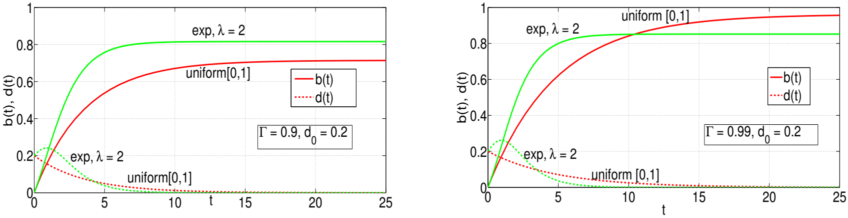
$$\begin{aligned} b(t) &= \frac{d_0}{r} - \frac{d_0}{r} e^{-rt} \quad (5) \\ d(t) &= d_0 e^{-rt} \quad (6) \end{aligned}$$

- **Exponential distribution with parameter λ:** $h_F(x) = \lambda$ (memoryless). The fluid limit is the solution to the o.d.e.,

$$\begin{aligned} \dot{b} &= d \\ \dot{d} &= \lambda \Gamma d(1 - b - d) - d \end{aligned}$$

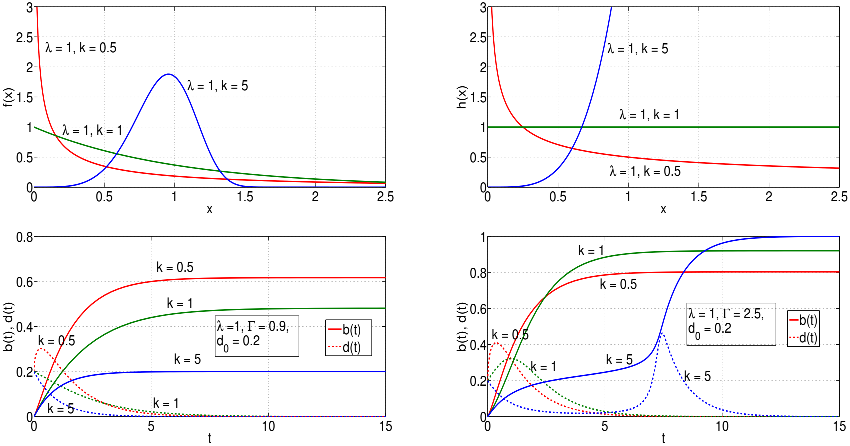
This is equivalent to the SIR epidemic model with infection rate $\lambda\Gamma$ and recovery rate of 1

Comparison: Uniform vs Exponential



- small Γ: exponential case yields a larger terminal influence spread
  - higher slope near zero; more nodes with threshold close to zero
- large Γ: uniform distribution yields a larger terminal influence spread
  - thresholds bounded above by 1

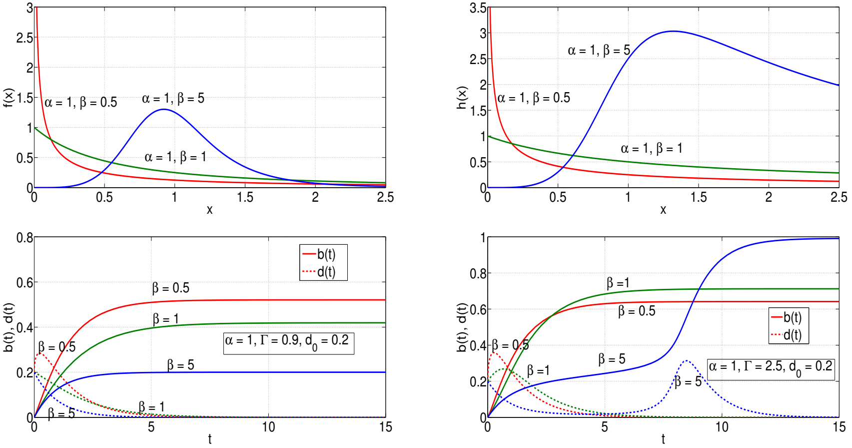
Weibull distribuion



$$f(x : \lambda, k) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{(k-1)} e^{-(\frac{x}{\lambda})^k}, \ \ x \geq 0$$

$$h(x; \lambda, k) = \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{(k-1)}, \ \ x \geq 0$$

Loglogistic distribuion



$$f(x;\alpha,\beta) = \frac{(\frac{\beta}{\alpha})(\frac{x}{\alpha})^{\beta-1}}{(1+(\frac{x}{\alpha})^{\beta})^2},\ x \geq 0$$

$$h(x;\alpha,\beta) = \frac{\beta}{\alpha}\left[\frac{(\frac{x}{\alpha})^{\beta-1}}{1+(\frac{x}{\alpha})^{\beta}}\right],\ x \geq 0$$

Multiclass HILT model

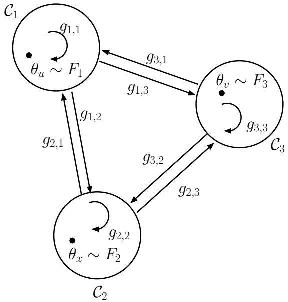


Figure: Heterogeneous network with three communities, shown with entries of the influence matrix $\mathcal{G}$. Nodes u, x and v belong to communities $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$, and have their thresholds distributed according to $F_1(\cdot)$, $F_2(\cdot)$ and $F_3(\cdot)$ respectively.

$$\dot{b}_i = d_i$$

$$\dot{d}_i = -d_i + [\mathcal{G}^T \mathbf{d}]_i \; h_{F_i}([\mathcal{G}^T \mathbf{b}]_i)(n_i - b_i - d_i)$$

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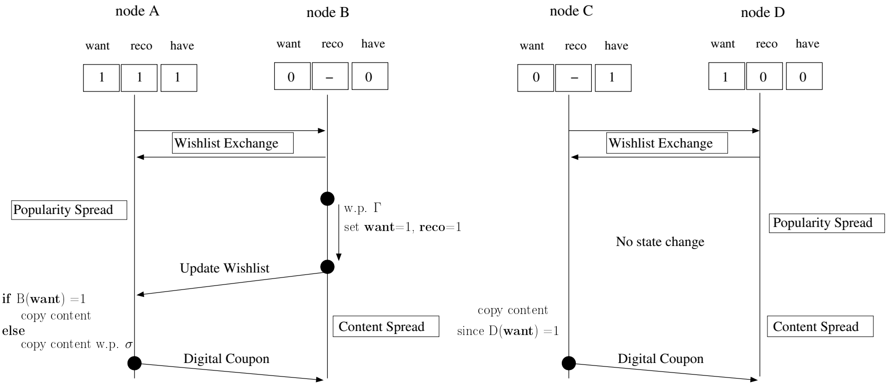
Mobile Opportunistic Networks (MON)

- Proliferation of smart mobile devices
  - Smartphones, tablets, wearable computing devices, etc.
- Mobile content delivery - a challenge
  - Direct delivery - not scalable
  - Alternative: p2p delivery (via Bluetooth, WiFi-Direct, etc.)
- A single item of content may be of **interest** to several co-located users
  - e.g., slides of a conference keynote/ course lecture
- The item can be forwarded between devices when their p2p radio interfaces make contact
  - Akin to **epidemic** spread
  - A multi-hop opportunistic mobile network
  - Provides an approach to *delay tolerant* networking

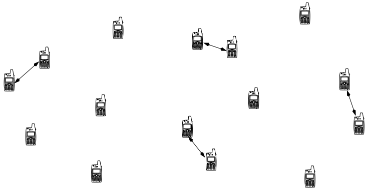
Mobile Coupon Delivery Ecosystem

- **Scenario:** Pre-release promotions of a product (movie, book, etc.)
- **Content:** discount coupon for pre-ordering the product
- Mobile users can
  - maintain a wishlist of products of interest
  - share wishlists with peers
  - receive coupons from peers or central server
- wishlists + peer recommendation + coupon delivery

A Possible Application Framework



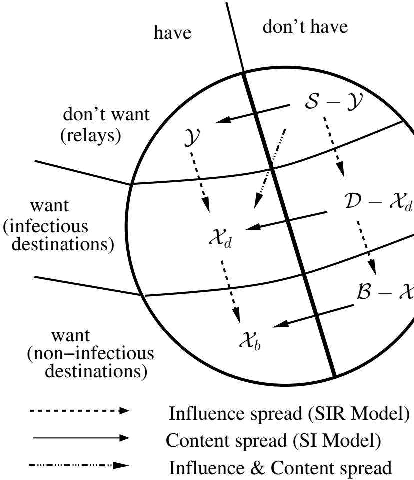
Content Popularity and Dissemination: SIR-SI model



- Population size: N (fixed)
- Pairwise meetings at points of independent Poisson processes
- The coupon needs to reach certain *destination* nodes
- Some destination nodes are given the coupon initially
- The set of destination nodes grows, as more nodes express their interest
- *Do-not-want* nodes could help in forwarding (relays)

Objective: Quickly spread content to a large fraction of destinations nodes while minimizing the residual number of relays that have the content

SIR-SI States and Evolution



- λ_N: Poisson meeting rate for each pair of nodes; $\lambda_N = \frac{\lambda}{N}$
- β_N: Recovery rate of infectious destinations; $\beta_N = \beta$
- Γ: Influence probability
- σ: Copying probability to a relay (control)

SIR-SI CTMC Markov Chain: Transitions at Meeting Epochs

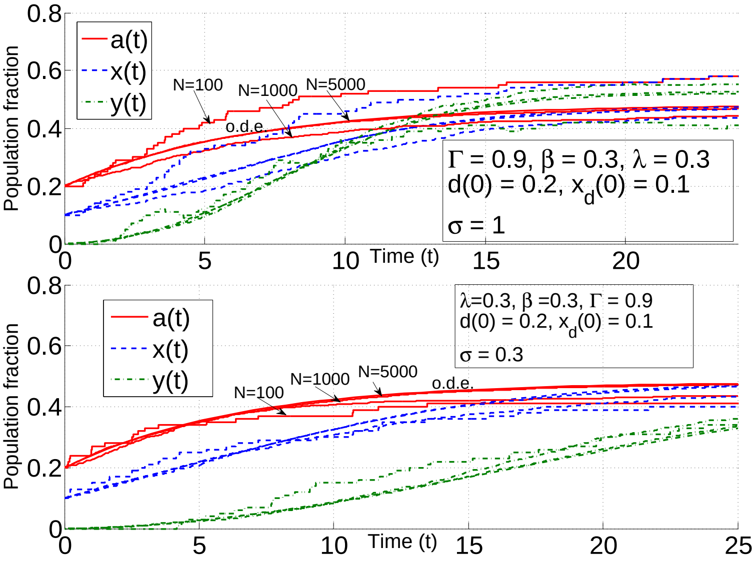
- Let $k = 0, 1, 2, \cdots$ index the meeting/recovery epochs at times t_k
- System state: $Z(k) = (B(k), D(k), X_b(k), X_d(k), Y(k))$

Epoch type	Rate	State update δ_k
$\mathcal{D} - \mathcal{X}_d$ recovers	$\beta_N(D(k) - X_d(k))$	(1,-1,0,0,0)
$\mathcal{X}_d$ recovers	$\beta_N X_d(k)$	(1,-1,1,-1,0)
$\mathcal{B} - \mathcal{X}_b$ meets $\mathcal{X} + \mathcal{Y}$	$\lambda_N(B(k) - X_b(k))(X(k) + Y(k))$	(0,0,1,0,0)
$\mathcal{D} - \mathcal{X}_d$ meets $\mathcal{Y}$	$\lambda_N(D(k) - X_d(k))Y(k)$	(0,0,0,1,0) + (0,1,0,1,-1) w.p. Γ
$\mathcal{D} - \mathcal{X}_d$ meets $\mathcal{X}$	$\lambda_N(D(k) - X_d(k))X(k)$	(0,0,0,1,0)
$\mathcal{X}_d$ meets $\mathcal{Y}$	$\lambda_N X_d(k)Y(k)$	(0,1,0,1,-1) w.p Γ
$\mathcal{S} - \mathcal{Y}$ meets $\mathcal{X}_b + \mathcal{Y}$	$\lambda_N(X_b(k) + Y(k))(S(k) - Y(k))$	(0,0,0,0,1) w.p. σ
$\mathcal{S} - \mathcal{Y}$ meets $\mathcal{D} - \mathcal{X}_d$	$\lambda_N(D(k) - X_d(k))(S(k) - Y(k))$	(0,1,0,0,0) w.p. Γ
$\mathcal{S} - \mathcal{Y}$ meets $\mathcal{X}_d$	$\lambda_N(X_d(k))(S(k) - Y(k))$	(0,1,0,1,0) w.p. Γ (0,0,0,0,1) w.p$(1 - \Gamma)\sigma$

$$
\begin{aligned}
\dot{b} &= \beta d \\
\dot{d} &= -\beta d + \lambda \Gamma d s \\
\dot{x}_b &= \beta x_d + \lambda(b - x_b)(x + y) \\
\dot{x}_d &= \lambda(d - x_d)(x + y) + \lambda \Gamma d y + \Gamma \lambda x_d(s - y) - \beta x_d \\
\dot{y} &= -\Gamma \lambda d y + \lambda \sigma (s - y)(x_b + y + (1 - \Gamma) x_d)
\end{aligned}
$$

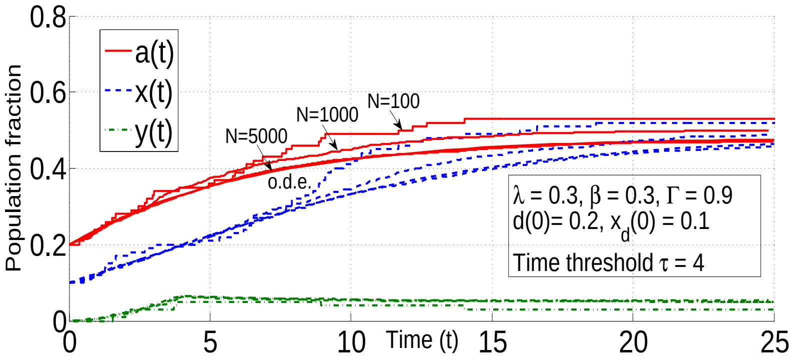
where $s(t) = 1 - b(t) - d(t)$

Convergence of the CTMC to O.D.E. Limit



SIR-SI Model: Dynamic Control of Copying

- Dynamic control $\sigma : Z(k) \to [0, 1]$
- CTMDP for each N: Obtaining optimal control is difficult
- Replace probabilistic control σ in the ODE by $\sigma(t)$ (controlled ODE)
- Optimal (deterministic, open-loop) control for the controlled ODE
- Can be shown to be asymptotically optimal for the finite size problem



Optimal Control

Target time: $T_\sigma = \inf\{t : x_\sigma(t) \geq \alpha a(\infty)\}$

- $a(\infty) = b(\infty) + d(\infty)$: terminal fraction of destinations
- $x_\sigma(t)$: fraction of destinations that have the coupon at time t

Cost function: $C_\sigma = \psi y_\sigma(T_\sigma) + T_\sigma = \psi y_\sigma(T_\sigma) + \int_0^{T_\sigma} 1 dt$

- $y_\sigma(T_\sigma)$: fraction of relays that have the coupon at time T_σ

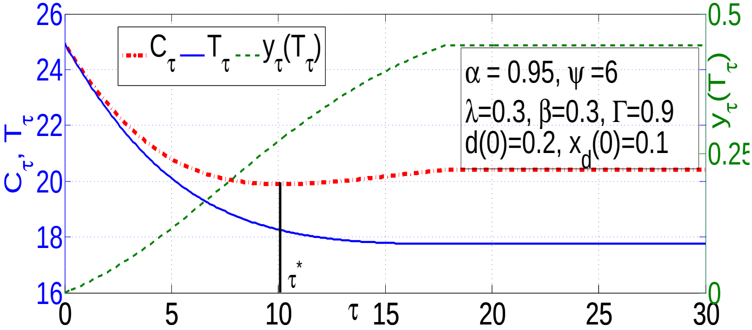
Theorem

For the above o.d.e. system, for the cost function displayed earlier, there exists an optimal control of the form,

$$\sigma_\tau(t) = \begin{cases} 1, & 0 < t < \tau \\ 0, & t \geq \tau \end{cases}$$

- Distributed implementation: Time stamping of the coupon

Optimal Control for the Running Example



- $\alpha = 0.95$, $C_\sigma = 6y_\sigma(T_\sigma) + T_\sigma$
- The optimal control is to copy until $\tau = 10.1$ and then stop copying

Outline

1 Introduction

2 Influence Maximization: Linear Threshold Model

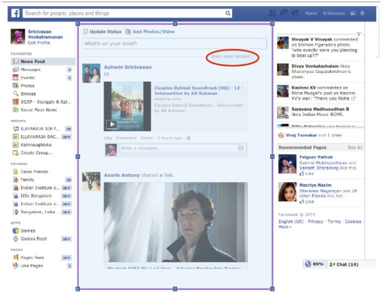
3 Fluid Limit Characterization: Linear Threshold Model

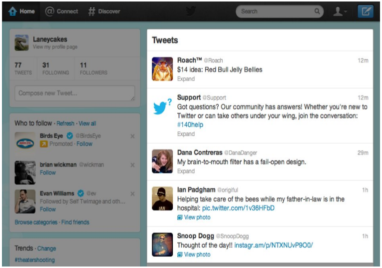
4 Coevolution of Influence and Content in Mobile P2P Networks

5 Competition over Timelines

Timelines on Social Networks

- Facebook, Twitter use a ***Timeline*** based social feed
  - **Reverse chronological** – latest entries pushing out older entries
  - Similar to an email inbox
- Google+, Pinterest employ **parallel timelines**
- Recently, OSNs also **sort the entries** according to user preference
  - Priority Inbox, Facebook's EdgeRank, etc.



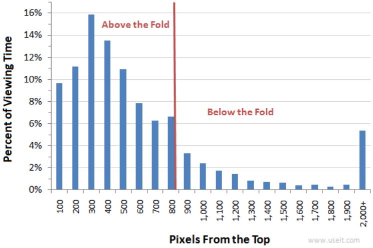


Social Advertising

- **Traditional online ads:** sponsored search, featured links, banner ads
  - Consumers are becoming more immune; Ads cannot be shared
- **Social ads:** Customized suggestions based on personal/social history
  - Brands have their own pages/accounts on the social network
  - Consumers can share or retweet the promotional content

Limited User Attention

- 80 % of the users' viewing time is spent on the contents *above the fold*
- Timeline: *User attention is limited to the top few items*

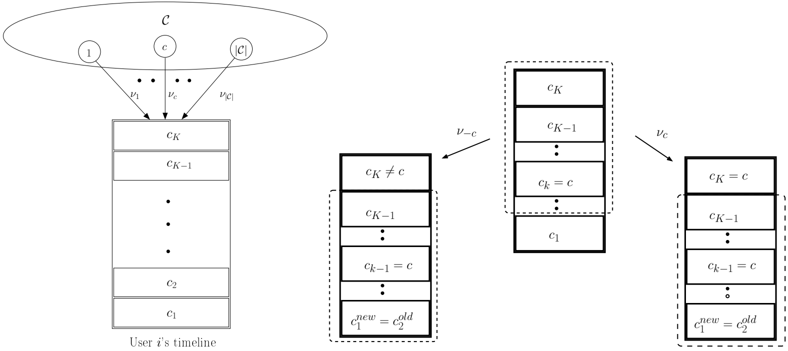


- Economy of user attention [Wu-Huberman 2007], Clash of the contagions [Myers-Leskovec 2012], Timeline model [Weng et al. 2012]
- Competition between content creators for timeline space yet to be analytically modeled

Source: http://www.nngroup.com/articles/scrolling-and-attention/

User's Timeline

- Reverse chronological timeline of size $K(i)$
- Items of content c generated at points of a Poisson process of rate ν_c
- $\nu := \sum_c \nu_c$, $\nu_{-c} := \sum_{c' \neq c} \nu_{c'}$



Stationary distribution: $\pi_{\mathbf{c}} = \prod_{k=1}^{K} \frac{\nu_{c_k}}{\nu}$

Expected *ic*-Busy Period

ic-busy period: The duration for which *at least one item* of c's content is present in user i's timeline *after* first entering the head of the timeline

$\alpha_c := \frac{\nu_{-c}}{\nu}$

Theorem

The expected ic-busy period is given by

$$E[T_{ic}(K)] = \frac{1}{\nu_c}\left(\frac{1-\alpha_c^{-K}}{1-\alpha_c^{-1}}\right)$$

Proof sketch: Recursive equation for $E[T_{ic}(k)]$, the duration for which content c stays on user i's timeline, starting at position k.

$$E[T_{ic}(0)] = 0,$$

$$E[T_{ic}(k+1)] = \frac{1}{\nu} + \frac{\nu_c}{\nu}E[T_{ic}(K)] + \frac{\nu_{-c}}{\nu}E[T_{ic}(k)]$$

Probability of Finding Content c on a User's Timeline

$$p_{ic} = \frac{E[T_{ic}(K)]}{E[T_{ic}(K)] + 1/\nu_c} = 1 - \alpha_c^K$$

Game formulation:

- **Players**: Content creators $c \in \mathcal{C}$, $|\mathcal{C}| = N$
- **Strategies**: $\nu_c \in [\phi, \infty)$, $\forall c \in \mathcal{C}$
- **Utility of player** c: $p_{ic} - \gamma_c\,(\nu_c - \phi)$
  - Models payment to the social network for additional promotion

$$\begin{array}{ll} \textbf{max} & 1 - \alpha_c^K - \gamma_c\,(\nu_c - \phi) \\ \textbf{s.t.} & \nu_c \geq \phi\ (\geq 0) \end{array}$$

Best response rate for content creator c obtained by solving for ν_c in

$$\nu_c = \nu \left[1 - \left(\frac{\gamma_c \nu}{K} \right)^{\frac{1}{K}} \right]$$

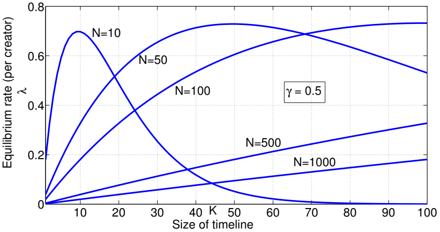
if this solution $> \phi$, else best response is ϕ

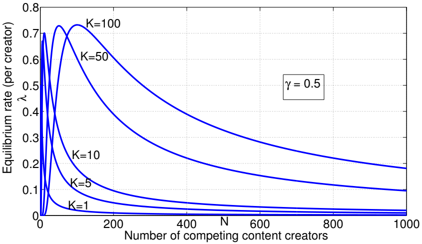
Equilibrium rate

Theorem

In a symmetric game ($\gamma_c = \gamma, \forall c \in \mathcal{C}$), restricting to equal rate profiles, at equilibrium each player sends either $\nu_c = \frac{K}{\gamma}\frac{(N-1)^K}{N^{(K+1)}}$ or ϕ, whichever is larger.

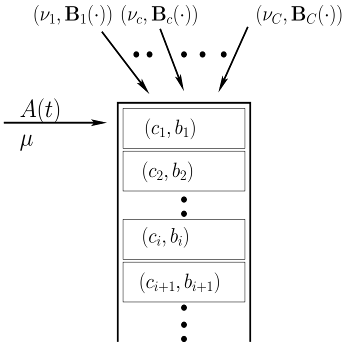
- For large N this is maximised when $\frac{K}{N} \approx 1$
- When $K << N$, too much competition $\Rightarrow$ use small ν
- When $K >> N$, little competition $\Rightarrow$ use small ν





Infinite Timeline Model: Content Generation and Influence

- Timeline size not restricted to K
- Each content creator is characterized by
  - **content generation rate** ν_c
  - **influence weight distribution** $B_c(\cdot)$
- User **visits timeline at points of a Poisson process** of rate μ
- On each visit, number of posts seen is **geometrically distributed**
  - stops at position j , $j \geq 1$ with prob. $\alpha^{j-1}(1-\alpha)$



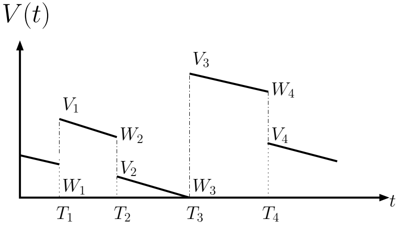
- Expected influence of content c is $v_c = \sum_{j=1}^{\infty} \mathbb{I}_{\{c_j=c\}} b_j \alpha^{j-1}$
- **Influence of the timeline at** t**:** $V(t)$

Evolution of Expected Influence, $V(t)$

- Focus on single content case
- Jumps in $V(t)$ occur only at the content arrival instants $T_k, k \geq 1$

$$V(T_k+) = b + \alpha V(T_k)$$

- Between these instants we assume that the value of $V(t)$ *decreases at a constant rate* (1 after appropriate scaling of time)



- Average influence on the user over the visits to the timeline
$$\lim_{t\to\infty} \frac{1}{A(t)} \int_0^t V(u)\mathrm{d}A(u)$$

Existence of Stationary Distribution, $F(\cdot)$

$V(t)$ is a piecewise deterministic Markov process. If $V(t)$ is asymptotically stationary and ergodic, the earlier limit can be obtained *a.s.* as the expectation w.r.t the stationary distribution

Theorem

Let $0 \leq \alpha < 1$ *and* $B(\cdot)$ *be supported on* $[0, b_{\max}]$. *Then, for the process* $V(t)$, *there exists a probability distribution* $F(\cdot)$ *on* $[0, \frac{b_{\max}}{1-\alpha})$, *such that,* $\forall v \in [0, \frac{b_{\max}}{1-\alpha})$

1. *Almost surely,*

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t I_{\{V(u) \leq v\}} \mathrm{d}u = F(v)$$

2. *Almost surely,*

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t V(u) \mathrm{d}u = \int_0^\infty (1 - F(u)) \mathrm{d}u$$

Level Crossing Analysis for $F(\cdot)$

We established existence of $F(\cdot)$: point mass p_0; density $f(x), x \in [0, \frac{b_{\max}}{1-\alpha})$
Using level-crossing analysis[Brill-Posner 1977], we obtain

$$f(x) + \nu \int_x^{\frac{x}{\alpha}} B(x - \alpha y) f(y) \mathrm{d}y = \nu p_0 B^c(x) + \nu \int_0^x B^c(x - \alpha y) f(y) \mathrm{d}y$$

- LHS: Downcrossing rate of level x
  - unit rate of decay with time
  - arrival of new content with influence that does not compensate for the α discount
- RHS: Upcrossing rate of level x
  - arrival of new content which sees the influence process at level 0 (with prob p_0), or
  - at level y, so that the incoming influence large enough to cause an upcrossing of x
- Taking Laplace transforms across the integral equation yields

$$\tilde{f}(s)(s - \nu) + \nu \tilde{b}(s) \tilde{f}(\alpha s) = \nu p_0 (1 - \tilde{b}(s))$$

Moments of the Stationary Distribution

$$
\begin{aligned}
p_0 &= 1 - \nu EB + \nu(1-\alpha)EV \\
&= 1 - \nu(1-\alpha)\left(\frac{EB}{1-\alpha} - EV\right)
\end{aligned}
$$

where EV is the expectation of $F(\cdot)$. Similarly, we can get

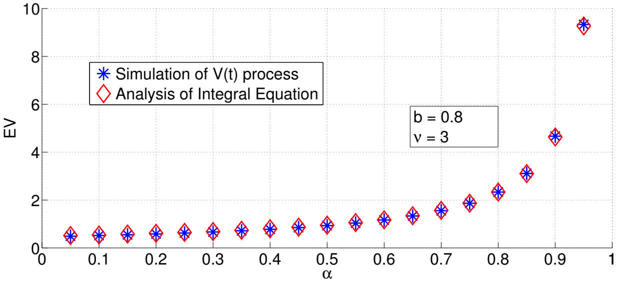
$$
EV = \frac{\nu EB^2 - \nu(1-\alpha^2)EV^2}{2(1-\alpha\nu EB)} \tag{7}
$$

When $\alpha = 1$, the process resembles the work-in-system of an $M/G/1$ queue

- p_0 equals the probability of finding the queue empty
- EV equals the expected work-in-system
- $V(t)$ can be thought of as work-in-system of an $M/G/1$ queue with *immediate discount at arrivals*

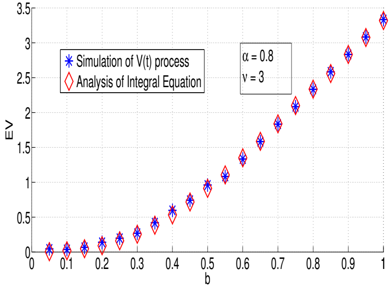
Expected Influence EV - Effect of α

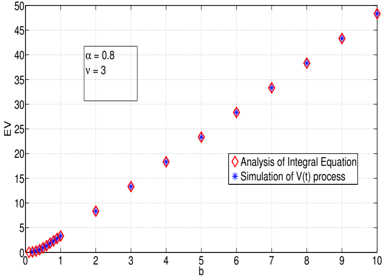
- Single publisher; content generation rate ν
- Each item brings a fixed influence b
- Recall that α is the parameter of the geometrically distributed number of timeline entries seen by the user on each visit



- For α close to zero, only the top entry's influence matters
- As α increases, the number of entries seen by the user increases, thus increasing the expected influence of the content

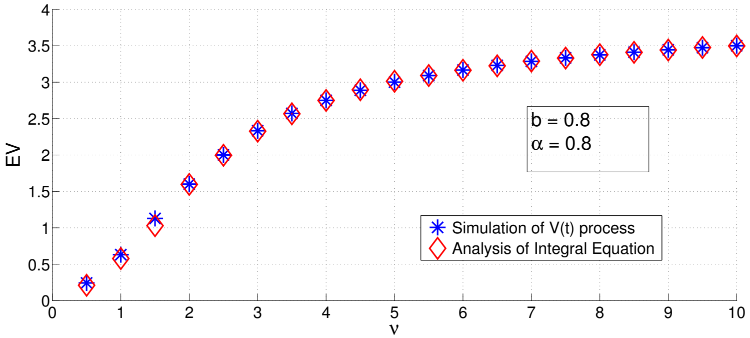
Expected Influence EV - Effect of b





- b, the influence weight is indicative of **quality**
- Increasing marginal returns for small ranges of b
  - Increasing b offsets the effect of decrease of influence with time
- As b increases, $p_0 \to 0$
- As p_0 approaches 0, the effect of b on EV becomes linear (with slope $\frac{b}{1-\alpha}$)

Expected Influence EV - Effect of ν



- ν, the content generation rate is indicative of **quantity**
- Diminishing marginal returns for increasing values of ν
- EV approaches $\frac{EB}{1-\alpha}$ as $\nu \to \infty$

Expected Influence: Multiple Content Case

- $\mathcal{C}$, the set of content creators with rates ν_c
- Define $\nu := \sum_{c\in\mathcal{C}} \nu_c$ and $\nu_{-c} = \nu - \nu_c$
- $V_c(t)$: the average influence process for content c
- Whenever a content $c' \neq c$ arrives, the net influence of content c is scaled down by α
  - From the perspective of content c, this is equivalent to an arrival with 0 influence
  - Reduce the analysis to the single content case, by introducing point mass at 0 in the arrival influence distribution
- If $b_c(x)$ is the probability density function of arrival influence of content c, then the modified distribution would be

$$\frac{\nu_{-c}}{\nu}\delta(x) + \frac{\nu_c}{\nu} b_c(x)$$

where $\delta(x)$ indicates point mass at 0

Level Crossing Analysis: Multiple Content Case

Integral equation for $f_c(\cdot)$

$$\begin{aligned} f_c(x) + \nu_{-c} \int_x^{\frac{x}{\alpha}} f_c(y)\mathrm{d}y + \nu_c \int_x^{\frac{x}{\alpha}} B_c(x - \alpha y) f(y)\mathrm{d}y \\ = \nu_c p_{c,0} B_c^c(x) + \nu_c \int_0^x B_c^c(x - \alpha y) f_c(y)\mathrm{d}y \end{aligned}$$

On taking Laplace transform:

$$\tilde{f}_c(s)(s - \nu) + (\nu_c \tilde{b}_c(s) + \nu_{-c})\tilde{f}(\alpha s) = \nu_c p_{c,0}(1 - \tilde{b}_c(s))$$

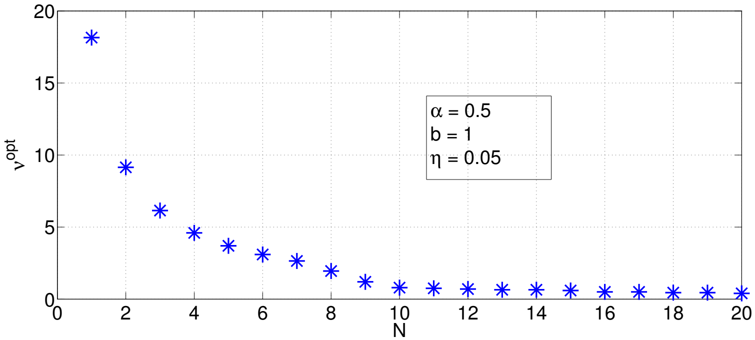
Expressions for $p_{c,0}$ and EV_c

$$p_{c,0} = 1 - \nu_c EB_c + \nu(1 - \alpha)EV_c$$

$$EV_c = \frac{\nu_c EB_c^2 - \nu(1 - \alpha^2)EV_c^2}{2(1 - \alpha\nu_c EB_c)}$$

N-Player Symmetric Game: Optimal rate vs. N

$$U_i = EV_i - \eta_i \nu_i b_i$$



- Equilibrium rate decreases monotonically with number of players N
- Contrast with finite timeline case (with a different objective function), where the behavior was non-monotonic with N

N-player symmetric game: Optimal weight

$$U_i = EV_i - \eta_i \nu_i b_i$$

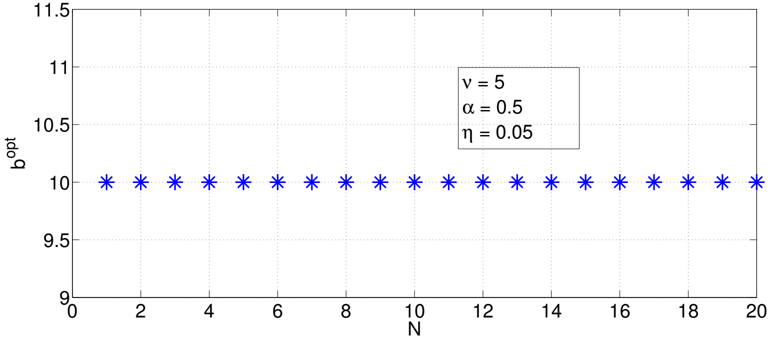
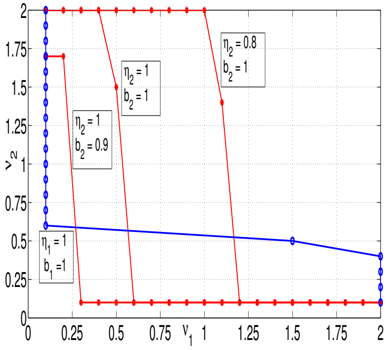


Figure: Variation of optimal influence weight (N-player symmetric game) with number of players N. .

- In this example, $b \in [0.1, 10]$

Two-Player Rate Competition: Examples of Equilibria



$$U_1 = EV_1 - \eta_1 b_1 \nu_1$$
$$U_2 = EV_2 - \eta_2 b_2 \nu_2$$

- Best response curves of player 1 (blue) and player 2 (red) for various values of (η_2, b_2) and $(\eta_1 = 1, b_1 = 1)$
- In this example, $\{\nu_{\min}, \nu_{\max}\} = \{0.1, 2\}$

- If $\eta_1 = \eta_2$ and $b_1 = b_2$, the equilibrium is symmetric $\nu_1^* = \nu_2^*$
- Decrease in 2's cost rate, η_2, allows use of larger ν_2
  - Causes player 1 to use higher content generation rate ν_1 at equilibrium
- Decrease in 2's influence, b_2, causes player 2 to use lower ν_2
  - Permits player 1 to use lower content generation rate ν_1 at equilibrium

Summary

Influence maximization

- Analytical characterization of LT model
- Optimal seeding set for simple networks
- Interpretation via APP in Markov chains
- G1-sieving algorithm

Fluid limit characterization

- Fluid limit o.d.e. for HILT model
- Equivalence to SIR epidemic model
- Effect of threshold distribution
- Fluid limit with heterogeneous communities

Coevolution of Interest and Content

- Application framework for coupon delivery
- SIR-SI model for the joint spread
- Kamke-dominance condition for o.d.e.s
- Delay-cost optimal time-threshold policy

Content Competition on Social Networks

- Competition over multiple social networks
  - Threshold structure of best response
- Competition over timelines
  - Finite timeline model
  - Infinite timeline model

Publications

Journals

- Srinivasan Venkatramanan and Anurag Kumar, *"Spread of Influence and Content in Mobile Opportunistic Networks,"* to appear in IEEE Transactions on Mobile Computing.
- Srinivasan Venkatramanan and Anurag Kumar, *"Fluid Limit Characterization of the Linear Threshold Model for Influence Spread on Social Networks,"* in preparation, 2014

Conference/Workshop Publications

- Srinivasan Venkatramanan and Anurag Kumar, *"Competition for Content Spread over Multiple Social Networks,"* Workshop on Social Networks in Science and Engineering (SCINSE'14), co-held with 6th Intl. Conference on Communication Systems and Networks (COMSNETS), Bangalore, India, 2014. (Recipient of the Best presentation award, sponsored by Adobe Research)
- Eitan Altman, Parmod Kumar, Srinivasan Venkatramanan and Anurag Kumar, *"Competition over Timeline in Social Networks,"* Workshop on Social Network Analysis and Algorithms (SNAA), co-held with IEEE/ACM ASONAM, Niagara Falls, Canada, 2013.
- Srinivasan Venkatramanan and Anurag Kumar, *"Co-evolution of Content Popularity and Delivery in Mobile P2P Networks,"* IEEE Infocom 2012 mini-Conference, Orlando, FL, 2012.
- Srinivasan Venkatramanan and Anurag Kumar, *"Information Dissemination in Socially Aware Networks under the Linear Threshold model,"* National Conference on Communication(NCC), IISc. Bangalore, 2011.

Technical Reports

- Srinivasan Venkatramanan and Anurag Kumar,*"Co-evolution of Content Popularity and Delivery in Mobile P2P Networks,"* arXiv:1107.5851
- Srinivasan Venkatramanan and Anurag Kumar, *"New Insights from an Analysis of Social Influence Networks under the Linear Threshold model,"* arXiv:1002.1335

Thank you